Useful Limits for Understanding Growth

Orders of Growth

Below: $a,b>0$, $r>1$ and $t\in\mathbb{R}$ are constants.

Powers grow qualitatively faster than Logarithms:

$$\lim_{n\to\infty}\frac{\ln^a n}{n^b}=\lim_{x\to\infty}\frac{\ln^a x}{x^b}=0$$

Exponentials grow qualitatively faster than Logarithms:

$$\lim_{n\to\infty}\frac{\ln^a n}{r^n}=\lim_{x\to\infty}\frac{\ln^a x}{r^x}=0$$

Exponentials grow qualitatively faster than Powers:

$$\lim_{n\to\infty}\frac{n^b}{r^n}=\lim_{x\to\infty}\frac{x^b}{r^x}=0$$

$(n+t)^n$ doesn't grow qualitatively faster than n^n:

$$\lim_{n\to\infty}\frac{(n+t)^n}{n^n}=\lim_{x\to\infty}\frac{(x+t)^x}{x^x}=e^a$$

Growth under nth root

Below: $a>0$ and $t\in\mathbb{R}$ are constants.

$$\lim_{n\to\infty}a^{\frac{1}{n}}=1$$

$$\lim_{n\to\infty}n^{\frac{1}{n}}=1$$

$$\lim_{n\to\infty}(an+t)^{\frac{1}{n}}=1$$

$$\lim_{n\to\infty}(n!)^{\frac{1}{n}}=\infty \quad \& \quad \lim_{n\to\infty}\frac{1}{(n!)^{\frac{1}{n}}}=0$$